Collateral Quality and House Prices†

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Abstract

This paper studies the effects of shifts in collateral quality on house prices and the macroeconomy in a dynamic general equilibrium model with credit and housing collateral. Depending on whether information on collateral quality is produced, our model features endogenous regime switching between two lending regimes. Shocks to collateral quality and the associated regime switching effectively amplify movements in house prices relative to rents and are able to reconcile the joint dynamics of house prices, the price-rent ratio and output observed in the data. Our estimation shows that the collateral quality shocks account for approximately half of variations in house prices and the price-rent ratio and a fifth of variations in investment during the housing boom and the Great Recession.

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Introduction

Risks in collateral quality, among other factors, set the stage for the Great Recession in the U.S. From 2002 to 2007, loans backed by real estate, once unanimously regarded as secured, were revealed to be unsecured (The Financial Crisis Inquiry Report, 2011; Gorton and Metrick, 2012). Accompanied by low risks in collateral, house prices and the price-rent ratio climbed to unprecedented heights, which was followed by significant deterioration in the collateral quality and sudden collapses of housing markets. The large swings in housing market prices are depicted in Figure 1.

On the one hand, the fact that house prices and the price-rent ratio almost coincide with each other in Figure 1 has proved to be challenging to explain with a standard asset pricing approach (Kiyotaki, Michaelides, and Nikolov, 2011). On the other hand, studies on collateral financing usually assume that collateral, which can be seized and sold by lenders in cases of default, can effectively prevent lenders from suffering losses (Kiyotaki and Moore, 1997). The effects of such collateral constraints have been intensively investigated in the literature, but the effects of risky collateral quality have received far less attention.

In this study, we attempt to answer the following questions. How crucial are fluctuations in collateral quality to asset prices and the macroeconomy? Do these fluctuations help account for the joint dynamics of output, house prices and the price-rent ratio observed in the data? If so, by how much?

These questions are motivated by the notion that collateral quality may not be as perfect as typically assumed, not only due to possible information asymmetry between two trading parties, but also due to the considerable costs of information collection and the prohibitively high levels of sophistication and required skills to process and analyze the information. As Hughes (2010) reported in the Financial Times, “Much of regulators’ efforts have been focused on

\[\text{Insert Figure 1 Here.}\]

\[\text{These patterns are not unique to the US, as Knoll, Schularick, and Steger (2017) documented them in 14 advanced economies.}\]
pushing issuers to provide more information on individual loans ... However ... this is not a top priority and they were in fact more concerned with developing methods to analyze and compare cash flow data across different deals.” A lack of clear and precise information about underlying asset quality is of greater concern to loan makers than adverse selection. Lenders either spend resources to acquire information or make decisions with coarser information. This information friction, along with the fluctuations in collateral quality, adds extra macroeconomic consequences.

Our model is an infinite-horizon dynamic stochastic general equilibrium model with housing collateral. Houses have intrinsic value, as they provide housing services to households and serve as collateral for entrepreneurs. Entrepreneurs face idiosyncratic investment efficiency shocks and use their collateral to obtain external finance for investment. The key assumption in this paper is that houses can be either “good” or “risky.” Good houses have positive intrinsic value, while risky houses have positive intrinsic value with a probability, which is referred to as “collateral quality,” and zero value with the remaining probability. Buyers and sellers in house markets do not know the exact quality of each unit of risky houses and share the same knowledge about types of collateral.

We refer to shifts in collateral quality as collateral quality shocks. We show that this kind of shock can reconcile the patterns of output, house prices and the price-rent ratio observed in Figure 1. The intuition is as follows. The intrinsic value of houses implies that house prices consist of the sum of the present values of rents and the liquidity premium. The latter is associated with collateral constraints because houses can serve as collateral to relax borrowing limits and provide liquidity. Variations in collateral quality can affect house prices more strongly than rents via the following channels: (i) the probability that houses are good or not, (ii) the liquidity premium and (iii) regime switching, leading to similar responses of house prices and the price-rent ratio to collateral quality shocks in terms of magnitude.

Regime switching arises because of the above-mentioned information friction. Specifically, there could exist two lending regimes. In one regime, which we call the Information Sensitive (IS) regime, a lender pays an information acquisition cost to learn the exact quality of risky collateral and makes loans based on the acquired information. In the other regime, which
we call the Information Insensitive (II) regime, the lender incurs no information acquisition cost and makes a loan based on the appraised value of risky collateral. Endogenous regime switching then emerges as an equilibrium outcome depending on the collateral quality. If the collateral quality is higher than an endogenous threshold, the lender chooses the II regime to avoid the information acquisition cost; if the collateral quality is lower than that threshold, the equilibrium regime must be the IS regime. Consequently, regime switching can give rise to sudden changes and exacerbate volatilities of asset prices and macroeconomic variables.

In addition, we show that collateral quality shocks can rationalize the comovements among output, house prices and the price-rent ratio through expansion and contraction of aggregate liquidity in the economy. This implication makes this type of shocks more impactful than other kinds of shock discussed in the literature. For example, housing demand shocks cannot generate a positive correlation between house prices and the price-rent ratio and financial shocks cannot generate procyclical house prices. This suggests that collateral quality shocks are crucial in replicating the patterns in Figure 1.

We estimate this endogenous regime-switching model with Bayesian methods. The model contains six shocks: productivity shocks, housing demand shocks, labor supply shocks, collateral quality shocks, financial shocks and investment-specific technology (IST) shocks. The estimated size of collateral quality shocks is 4.8 times as large as productivity shocks by unconditional standard deviation. However, because of their counterfactual implication, housing demand shocks are estimated to be small, in contrast to the findings in the literature.

With the estimated parameters, the model-simulated real business cycle moments match those from the data reasonably well, especially regarding the large volatilities of investment, house prices and the price-rent ratio relative to output. In counterfactual experiments, we show that shutting down collateral quality shocks can reduce the volatilities of housing markets by up to half, and that shutting down housing demand shocks or financial shocks does not affect the volatilities substantially, echoing the above-mentioned implications.

Furthermore, we show that collateral quality shocks are quantitatively important to the boom and bust around the Great Recession. They account for more than half of the variations
in house prices and the price-rent ratio during the housing boom and the crisis, the largest share among all types of shocks. In addition to IST and other shocks, collateral quality shocks generate approximately one fifth of the variations in investment. Productivity shocks also matter, whereas housing demand shocks and financial shocks have minor effects. Moreover, although not targeted in the estimation of this study, the model-implied historical path of financial tightness replicates the data well.

Notably, the fitted path generated by the estimated model features an obvious regime switch at the start of the Great Recession. We conduct an additional test on the empirical fit of a reference model for which we shut down the regime switching. The log marginal densities of the data suggest that the data favors the benchmark model over the reference model. The identified regime switch per se causes additional drops in investment, consumption, house prices and the price-rent ratio, the magnitudes of which are non-negligible compared to the decreases in these variables throughout the whole crisis, corroborating the amplification effects of regime switching to asset pricing and macroeconomic volatilities.

**Literature review** First, this paper contributes to the growing literature on fluctuations in house prices and their macroeconomic consequences.² Studies in this literature examine housing collateral from various perspectives. Paying little attention to the price-rent ratio, Iacoviello (2005), Iacoviello and Neri (2010), Liu, Wang, and Zha (2013) and Guerrieri and Iacoviello (2017) attribute the excessive volatility in house prices mainly to widespread changes in housing demand. Kiyotaki, Michaelides, and Nikolov (2011) and Sommer, Sullivan, and Verbrugge (2013) study models with a house rental market, idiosyncratic risks and financing constraints in a small open economy, focusing on explaining the upward trend of house prices and the price-rent ratio during the housing boom rather than the Great Recession. Favilukis, Ludvigson, and Nieuwerburgh (2017) and Kaplan, Mitman, and Violante (2020) develop quantitative general equilibrium models and emphasize the roles of heterogeneous agents in propagating housing market prices. The former study argues for aggregate business cycle risks and bequest

²Davis and Nieuwerburgh (2015), Guerrieri and Uhlig (2016) and Piazzesi and Schneider (2016) give excellent surveys on this topic.
heterogeneity in preference as for an explanation for the house price movements, and the latter points to shifts in belief about future price appreciation. More recently, Justiniano, Primiceri, and Tambalotti (2019) and Liu, Wang, and Zha (2019) highlight the importance of variations in the credit supply on the housing boom and on the dynamics of the price-rent ratio, respectively. Garriga, Manuelli, and Peralta-Alva (2019) and Miao, Wang, and Zha (2020) reconcile the disconnect between house prices and rents by underscoring the role of segmentation in asset markets and discount shocks, respectively. This paper complements these studies by providing an alternative explanation for these issues and by exploring the endogenous interactions among collateral quality, house prices and the aggregate economy.

Second, this paper relates to studies on information opaqueness in financial markets and its consequences. A closely related work is Gorton and Ordoñez (2014), which also studies the two types of debt contracts, IS and II, and demonstrates that information opaqueness on collateral quality can give rise to both a credit boom and a subsequent collateral crisis. Asriyan, Laeven, and Martin (2021) also show that withholding information production can endogenize credit booms and resource misallocation. Although these papers and our paper all work on information opaqueness in financial markets, our paper differs from theirs in two important aspects. First, the collateral value is exogenously given in their papers, while the collateral value in our paper is endogenously determined, as we are interested in understanding the endogenous effects of collateral quality on the collateral value. Second, their papers aim for theoretical illustrations and are generally qualitative. Our paper builds on these studies and gauges whether and how shifts in collateral quality and the associated information friction can help to explain the data. In addition to the two papers, other works demonstrate that information opaqueness can enhance liquidity of assets in normal periods but may exacerbate market collapses in crisis periods (e.g., Pagano and Volpin, 2012; Hanson and Sunderam, 2013; Dang, Gorton, Holmström, and Ordoñez, 2017).

It is worth mentioning that our paper differs from the literature on asymmetric information and asset quality, such as Kurlat (2013), Guerrieri and Shimer (2014), Bigio (2015) and Asriyan, Fuchs, and Green (2019). The key mechanism in these studies is that, adverse selection acts as
a shadow cost for obtaining liquidity and generates feedback to the real economy, whereas our paper focuses on the aggregate effects of symmetric ignorance/awareness.

Finally, this paper is connected to the literature on the nonlinear effects of regime switching in financial markets. Mendoza (2010), Brunnermeier and Sannikov (2014), He and Krishnamurthy (2019), Benigno, Foerster, Otrok, and Rebucci (2020), among others, exemplify this strand of literature. Like these studies, we consider transitions between two regimes and explore the equilibrium behavior of crisis emergence and excessive asset price volatilities. However, unlike these studies, where endogenous regime switching typically arises with occasionally binding borrowing constraints, the regime switching in our model is associated with endogenous information production on collateral quality.

The remainder of this paper proceeds as follows. Section 2 describes the setup of the benchmark model. We solve the model and characterize the competitive equilibrium in Section 3. We quantitatively examine the model in Section 4 and further explore the mechanisms in Section 5. Section 6 concludes the paper.

2 Model

Consider a discrete-time and infinite-horizon real economy populated by a representative household. A household is a large family consisting of four types of members: workers, bankers, entrepreneurs and capital producers. Workers supply labor to production. Bankers provide loans to entrepreneurs. Entrepreneurs have skills for production and invest in capital. Capital producers create new capital goods and sell them to entrepreneurs. There are three types of goods in this economy: consumption goods, capital goods and houses.

2.1 Households

A household has a continuum of identical workers, a continuum of identical bankers, a continuum of identical capital producers and a continuum of entrepreneurs, all of whom are of a unit mass. In each period, the household consumes the following composite of consumption goods
\( C_t \) and housing services \( \bar{H}_t + \eta_t H_t \),

\[
X_t = \left[ (1 - \psi_{Ht}) C_t^\frac{1}{\chi} + \psi_{Ht} \left[ \exp(gt) (\bar{H}_t + \eta_t H_t) \right]^\frac{1}{\chi} \right]^\frac{\chi}{\chi-1},
\]

where \( \chi > 1 \) governs the elasticity of substitution between consumption goods and housing services, and \( \psi_{Ht} \in (0, 1) \) measures the utility weight on housing services, which reflects housing demand. There are two types of houses: good houses, denoted by \( \bar{H}_t \), each unit of which provides positive utility to households, and risky houses, denoted by \( H_t \), each unit of which is good with probability \( \eta_t \in (0,1) \) and bad with probability \( 1 - \eta_t \). Bad houses provide zero utility and can be thought as a lemon or a toxic asset.\(^3\) At the beginning of a period, all agents in this economy know the type (good or risky) and the probability \( \eta_t \), but they do not know the exact quality of each unit of risky houses. We assume that households do not rent their own houses and that there is no learning. At the end of a period, households pay rents only for good houses, but they cannot disclose information to house and rental markets.

Here, the utility weight \( \psi_{Ht} \) stands for a stochastic housing demand shock, as in Iacoviello (2005) and Liu, Wang, and Zha (2013). It follows an AR(1) process, \( \ln(\psi_{Ht}) = (1 - \rho_h) \ln(\psi_{Ht-1}) + \rho_h \ln(\psi_{Ht-1}) + \sigma_h \varepsilon_{Ht} \), where the persistence \( \rho_h \in (-1, 1) \) and the standard deviation \( \sigma_h > 0 \). \( \varepsilon_{Ht} \) is an independent and identically distributed (IID) standard normal random variable. The economy grows at a constant rate \( g > 0 \), and the term \( \exp(gt) \) ensures a balanced growth path with a non-growing supply of houses.

Each worker in this family supplies labor \( N_t \) in each period. The household maximizes its life-time utility

\[
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[ X_t - \psi_X X_{t-1} - \psi_{Nt} \exp(gt) N_{t}^{1+\nu} \right]^{1-\kappa} \left[ \frac{1}{1+\nu} \right],
\]

where \( \beta \in (0, 1) \) is the discount factor, \( \kappa > 0 \) measures the curvature of the period utility func-

\(^3\)In our context, \( \eta_t \) reflects the asset quality in terms of generating positive payoff flows, rather than construction quality as considered in Stroebel (2016). For a more general setup, the assumption that bad houses provide zero utility can be relaxed to that bad houses provide positive but lower utility than good houses.
tion, $\psi_X \in (0, 1)$ measures habit formation, and $\nu > 0$ captures the inverse of Frisch elasticity of labor supply. $\psi_N$ is the disutility weight on labor and represents a labor supply shock. It follows an AR(1) process $\ln(\psi_N_t) = (1 - \rho_n) \ln(\psi_N) + \rho_n \ln(\psi_N_{t-1}) + \sigma_n \varepsilon_{Nt}$ where the persistence $\rho_n \in (-1, 1)$ and the standard deviation $\sigma_n > 0$. $\varepsilon_{Nt}$ is an IID standard normal random variable.

The family pools the labor income $W_t N_t$ and all dividends (or profits) from entrepreneurs, bankers and capital producers $D^e_t + D^b_t + D^k_t$ together and distributes them equally to all members. Thus, the household’s budget constraint is given by

$$C_t + R_t (\overline{H}_t + \eta_t H_t) \leq W_t N_t + D^e_t + D^b_t + D^k_t,$$

where $R_t$ and $W_t$ denote rents and wages in real terms, respectively. In a nutshell, a household maximizes (2) by choosing proper $\{C_t\}_{t=0}^\infty$, $\{N_t\}_{t=0}^\infty$, $\{\overline{H}_t\}_{t=0}^\infty$ and $\{H_t\}_{t=0}^\infty$ subject to constraints (1) and (3). Let $\Lambda_t$ denote the Lagrangian multiplier associated with (3). The optimal decisions on $C_t$, $N_t$, $\overline{H}_t$ and $H_t$ satisfy

$$\Lambda_t = (1 - \psi_{Ht}) X_t^{\frac{1}{\xi}} C_t^{-\frac{1}{\xi}} \left[ \left( X_t - \psi_X X_{t-1} - \psi_{Nt} \exp(g t) N_t^{1+\nu} \right)^{-\kappa} \right],$$

$$W_t \Lambda_t = \left( X_t - \psi_X X_{t-1} - \psi_{Nt} \exp(g t) \frac{N_t^{1+\nu}}{1+\nu} \right)^{-\kappa} \exp(g t) \psi_{Nt} N_t^\nu,$$

$$\left( 1 - \psi_{Ht} \right) R_t C_t^{-\frac{1}{\xi}} = \psi_{Ht} [\exp(g t)]^\frac{\nu-1}{\xi} (\overline{H}_t + \eta_t H_t)^{-\frac{1}{\xi}}.$$
2.2 Entrepreneurs

An entrepreneur indexed by $j$ employs capital and labor as inputs and produces consumption goods $Y_{jt}$ with a Cobb-Douglas production function

$$Y_{jt} = K_{jt}^\alpha (A_t N_{jt})^{1-\alpha}, \quad (7)$$

where $A_t$, $K_{jt-1}$, and $N_{jt}$ represent aggregate productivity, capital input, and labor input, respectively. $\alpha \in (0, 1)$ is the capital share in production. Define $A_t = \exp(gt)a_t$ where $a_t$ is a transitory productivity shock. Assume that $a_t$ follows an AR(1) process $\ln(a_t) = \rho_a \ln(a_{t-1}) + \sigma_a \varepsilon_{at}$ where the persistence $\rho_a \in (-1, 1)$ and the standard deviation $\sigma_a > 0$. $\varepsilon_{at}$ is an IID standard normal random variable.

At the beginning of period $t$, each entrepreneur receives an idiosyncratic investment efficiency shock $\epsilon_{jt}$. With this shock, the capital stock of an entrepreneur $j$ evolves as

$$K_{jt} = (1-\delta)K_{jt-1} + \epsilon_{jt}I_{jt}. \quad (8)$$

Here, $\epsilon_{jt}$ is randomly drawn from a distribution with cumulative distribution function $\mathcal{F}(\epsilon)$. For tractability, we assume that $\epsilon_{jt}$ is IID across entrepreneurs and periods, and that investment is irreversible, i.e., $I_{jt} \geq 0$.

When an entrepreneur observes shock $\epsilon_{jt}$, he has not yet received his sales revenue in period $t$, so he has to rely on external financing to invest. He can use his houses $H_{jt-1}$ and $\widetilde{H}_{jt-1}$ as collateral to borrow from bankers, as houses, unlike goods, cannot be hidden from lenders.\footnote{Including physical capital as collateral does not substantially alter our results. We think of houses as assets distinct from physical capital so that the collateral value is orthogonal to firms’ transformation efficiency from investment to capital stock. This distinction is, of course, stark, but it is helpful to isolate the effects of fluctuations in the collateral value from those in productivity and aggregate investment-specific technology.}

Therefore, the collateral constraint for entrepreneur $j$ is given by

$$P_{kt}I_{jt} \leq \theta_t(\widetilde{P}_tH_{jt-1} + \widetilde{\widetilde{P}}_tH_{jt-1}), \quad (9)$$
where $P_{kt}$ is the price of capital goods, $P_t$ is the price of good houses, and $\tilde{P}_t$ is the collateral value per unit of risky houses, which is derived in the next section. As explained in Kiyotaki and Moore (1997) and Jermann and Quadrini (2012), in the case of default, for a given amount of collateral, the liquidation value is only a fraction $\theta_t < 1$ of its full value because of financial frictions. $\theta_t$ reflects financial tightness, with a higher value indicating larger loans for a given amount of collateral. It can be interpreted as a financial shock, similar to that in Jermann and Quadrini (2012), following an AR(1) process $\ln(\theta_t) = (1 - \rho_\theta) \ln(\theta) + \rho_\theta \ln(\theta_{t-1}) + \sigma_\theta \varepsilon_{\theta t}$ where the persistence $\rho_\theta \in (-1, 1)$ and the standard deviation $\sigma_\theta > 0$. $\varepsilon_{\theta t}$ is an IID standard normal random variable. Loans are intratemporal, and the entrepreneur pays the loan back at the end of period $t$.\(^5\)

Entrepreneur $j$ maximizes the sum of the expected present values of dividend payments

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta_t^t \Lambda_t^t D_{jt},$$

where $\beta_t^t \Lambda_t^t$ is the stochastic discount factor and the dividends $D_{jt}$ are given in Section 3.

Before deriving the endogenous collateral value $\tilde{P}_t$, we summarize the timing of events within a period.

1. All kinds of shocks are realized at the beginning of period $t$. Agents know the quality of risky houses $\eta_t$ without knowing which specific unit is good.
2. Entrepreneurs who decide to invest borrow from bankers.
3. Entrepreneurs invest, produce and repay their loans.
4. Good and risky houses are traded.
5. Households receive all earnings from their members and pay rents.

\(^5\)While complicating the algebra, allowing loans to be intertemporal does not fundamentally change the insights generated by the model.
2.3 Collateral quality and debt contract

We now derive the endogenous collateral value $\tilde{P}_t$. Bankers operate in a competitive market and make loans to entrepreneurs with collateral. If the face value of a loan is not repaid, the banker seizes the collateral and sells it on the house market. The borrower can re-enter the credit market without penalty in the next period. In this section, we omit financial shocks and consider a unit of collateral.

Being riskless, a unit of good houses supports a loan of size $\tilde{P}_t$, as shown in (9). For risky houses, the banker can learn their true quality at an information acquisition cost, a fraction $\gamma \in (0, 1)$ of the appraised collateral value. Once the information is produced, it goes public. Hence, once identified as bad, the houses will not be traded and automatically exit from all markets. If information is not produced, house types remain unchanged.

There are two types of debt contracts for risky collateral depending on whether or not the information is produced: IS and II debt. These two types of debt are similar to those in Gorton and Ordoñez (2014) but with a completely different focus. In their study, assuming the
collateral value to be exogenous and constant, the authors study how the strategic behavior of borrowers regarding their loan sizes affects the types of loans. In our study, by endogenizing house prices, we focus on how lending types and house prices interact with each other. We analyze these two types of debt below.

**Information Sensitive Debt**  Let $\tilde{P}_i^S$ denote the loan size per unit of risky collateral with an IS contract. With this type of contract, a lender pays the information acquisition cost and learns the true quality of the collateral. If the collateral is good, the debt contract is signed and the borrower obtains a loan of $\tilde{P}_i^S$; if the collateral is bad, the contract is not signed. As the credit market is competitive, a banker is indifferent between lending or not, i.e.,

$$\eta_t(P_t - \tilde{P}_i^S) = \gamma \eta_t \tilde{P}_t,$$

where the right-hand side of the equation is the information acquisition cost, and the left-hand side of the equation is the expected benefit of lending. If the collateral is good, by a no-arbitrage argument, the borrower pays $P_t$ back anyway, leaving the benefit to the lender as $P_t - \tilde{P}_i^S$; if the collateral is bad, the benefit to the lender is zero. Therefore, $\tilde{P}_i^S = (1 - \gamma) P_t$ which is the price of good houses net of the information acquisition cost.

**Information Insensitive Debt**  Let $\tilde{P}_i^I$ denote the loan size per unit of risky collateral with an II contract. With this type of contract, a lender does not acquire the information about the true quality of the collateral and extends the loan according to its appraised value. The lender’s participation constraint becomes

$$P_t - \tilde{P}_i^I = 0,$$

where $P_t$ is the price of risky houses. In this case, the cost of lending is simply zero, while the benefit of lending is $P_t - \tilde{P}_i^I$, because the lender always makes a loan of $\tilde{P}_i^I$ and the borrower always pays back $P_t$ by a non-arbitrage argument. It follows that $\tilde{P}_i^I = P_t$ which is the price of
risky houses.

However, this type of debt is not implemented unless the lender has no incentive to deviate. The lender may deviate if he finds it more profitable to secretly pay the information acquisition cost and learn the exact quality, while only lending against good collateral. In other words, this type of contract is implementable only if secret information production is unprofitable, i.e.,

\[ \eta_t (P_t - \bar{P}_t^I) < \gamma P_t. \]

The left-hand side of the equation is the expected gain from behaving as if the lender honors the II contract when the collateral is good, while the right-hand side of the equation is the cost of information production. Thus, the following lemma is straightforward.

**Lemma 1** The II debt contract can be implemented in the equilibrium only if

\[ \eta_t \left( \frac{P_t}{P_t} - 1 \right) \leq \gamma. \]  

(11)

If condition (11) does not hold, the equilibrium lending type can only be IS. The above lemma describes a necessary condition for II type to be implemented. To check whether (11) is sufficient, we have the following lemma.

**Lemma 2** If condition (11) holds, then

\[ P_t \geq (1 - \gamma) \eta_t \bar{P}_t. \]

Proof. Please see Appendix A.1. ■

Lemma 2 tells us that, when both types are possible, for a given unit of collateral, an II debt contract allows for a larger loan than an IS debt contract. In that case, the competitiveness of the credit market will drive the II type as the only type in equilibrium. To summarize, there are two lending regimes - the **II regime** and **IS regime** - with the determination of the equilibrium regime characterized by the following proposition:
Proposition 1 The II debt contract is implemented in the equilibrium if and only if condition (11) holds.

This proposition gives the necessary and sufficient conditions for the equilibrium regime: it is the II regime whenever (11) holds. Therefore,

\[
\bar{P}_t = \begin{cases} 
P_t, & \text{if condition (11) holds;} \\
(1 - \gamma)\bar{P}_t, & \text{if condition (11) does not hold.}
\end{cases}
\] (12)

Noting that house prices \( \bar{P}_t \) and \( P_t \) are endogenously determined, responding to the collateral quality \( \eta_t \) and other shocks, Proposition 1 describes an endogenous regime switching condition. Using this proposition, in Section 4, we show that our model estimation identifies an endogenous regime switch in the data.

We regard the collateral quality \( \eta_t \) as the collateral quality shock. Empirical evidence shows that collateral quality varies over time as a result of institutional or technological changes in financial markets (see Keys, Mukherjee, Seru, and Vig, 2010; Becker, Bos, and Roszbach, 2020). Many theoretical works have been written on the possible origins of these variations, and we take them as given throughout this paper. We assume that \( \eta_t \) follows an AR(1) process \( \ln(\eta_t) = (1 - \rho_\eta)\ln(\eta) + \rho_\eta\ln(\eta_{t-1}) + \sigma_\eta\epsilon_{\eta t} \) where \( \eta \) is the unconditional mean of \( \eta_t \), \( \rho_\eta \in (-1, 1) \) measures the time persistence and \( \sigma_\eta > 0 \) measures the standard deviation. \( \epsilon_{\eta t} \) is an IID standard normal random variable. In Section 3, we demonstrate that \( \eta_t \) affects house prices not only directly, but also indirectly by affecting regime switching.

2.4 Capital producers

A representative capital producer uses consumption goods as inputs and produces new capital goods subject to adjustment costs. The capital producer sells the new capital goods in a competitive market at the price \( P_{kt} \). A capital producer chooses \( \{I_t\}_{t=0}^\infty \) to solve

\[
\max E_0 \sum_{t=0}^{\infty} \beta^t \frac{\Lambda_t}{\Lambda_0} \left\{ P_{kt} I_t - \left[ 1 + \frac{\Omega}{2} \left( \frac{I_t}{I_{t-1}} - \exp(g) \right) \right]^2 \frac{I_t}{Z_t} \right\},
\] (13)
where $\Omega > 0$ represents the adjustment costs, and $Z_t$ represents an aggregate investment-specific technology (IST) shock, as in Greenwood, Hercowitz, and Krusell (1997). This shock affects the aggregate technology of transforming investment into physical capital, and follows an AR(1) process $\ln(Z_t) = \rho_z \ln(Z_{t-1}) + \sigma_z \varepsilon_{Zt}$, where the persistence $\rho_z \in (-1, 1)$ and the standard deviation $\sigma_z > 0$. $\varepsilon_{Zt}$ is an IID standard normal random variable. The optimal investment $I_t$ satisfies

$$Z_t P_{kt} = 1 + \frac{\Omega}{2} \left( \frac{I_t}{I_{t-1}} - \exp(g) \right)^2 + \Omega \left( \frac{I_t}{I_{t-1}} - \exp(g) \right) \frac{I_t}{I_{t-1}} - \beta \mathbb{E}_t \Lambda_{t+1} + \frac{1}{\Lambda_t} \Omega \left( \frac{I_{t+1}}{I_t} - \exp(g) \right) \frac{Z_t}{Z_{t+1}} \left( \frac{I_{t+1}}{I_t} \right)^2. \tag{14}$$

3 Competitive Equilibrium

We now solve an entrepreneur’s problem and characterize the competitive equilibrium.

3.1 An entrepreneur’s solution

We start with an entrepreneur $j$’s choice of labor input. This choice is a static problem as follows

$$\max_{N_{jt}} K_{jt-1}^{a} (A_t N_{jt})^{1-a} - W_t N_{jt}. \tag{15}$$

The maximum of the above objective is $R_{kt} K_{jt-1}$, where $R_{kt}$ is the marginal product of capital and equal to

$$R_{kt} = \alpha \left[ \frac{(1 - \alpha) A_t}{W_t} \right]^{\frac{1-a}{\alpha}}. \tag{16}$$

We then study the entrepreneur’s dynamic problem. Let $\overline{H}_{jt} \in [0, H_{jt-1}]$ and $\overline{H}_{jt} \in [0, H_{jt-1}]$ denote the good and risky houses collateralized by the entrepreneur, respectively. Let $\delta_h \in (0, 1)$ denote the depreciation rates of both types of houses. No matter how much $\overline{H}_{jt}$ is, after repaying his loan, the entrepreneur is always left with a value of $(1 - \delta_h) P_t \overline{H}_{jt} + (1 - \delta_h) P_t \overline{H}_{jt}$.
The entrepreneur chooses $\delta_h)P_t(\overline{H}_{jt-1} - \overline{H}_{jt}) = (1 - \delta_h)P_t\overline{H}_{jt-1}$ for his good houses. Similarly, in the II regime, the entrepreneur is always left with a value of $(1 - \delta_h)P_tH_{jt} + (1 - \delta_h)P_t(\overline{H}_{jt-1} - \overline{H}_{jt}) = (1 - \delta_h)P_tH_{jt-1}$ for his risky houses. However, in the IS regime, after repaying his loan, the entrepreneur is left with a value of $(1 - \delta_h)(1 - \gamma)\eta_tP_tH_{jt}$ for his collateralized risky houses and a value of $(1 - \delta_h)P_t(H_{jt-1} - H_{jt})$ with for uncollateralized risky houses.

The entrepreneur’s dividends are his revenue from the capital income $R_{kt}K_{jt-1}$ and house rental income $R_t(\overline{H}_{jt} + \eta_tH_{jt})$ net of his expenditure on investment $P_{kt}I_{jt}$ and house purchases $P_t\left[(1 - \delta_h)\overline{H}_{jt-1} + \overline{H}_{nt} - \overline{H}_{jt}\right]$ and $1_5^\delta(1 - \delta_h)\left[(1 - \gamma)\eta_tP_tH_{jt} + P_t(H_{jt-1} - H_{jt})\right]$ + $(1 - 1_5^\delta)(1 - \delta_h)P_tH_{jt-1} + P_tH_{nt} - P_tH_{jt}$, where the indicator variable $1_5^\delta$ denotes the equilibrium lending regime governed by Proposition 1,

$$1_5^\delta = \begin{cases} 0, & \text{if II regime;} \\ 1, & \text{if IS regime.} \end{cases}$$ (17)

Here, $\overline{H}_t^n$ and $H_t^n$ are new good and risky houses, respectively, emerging in each period. They are universal and taken as given by all entrepreneurs. The entrepreneur’s dividends $D_{jt}$ are then given by

$$D_{jt} = R_{kt}K_{jt-1} - P_{kt}I_{jt} + R_t(\overline{H}_{jt} + \eta_tH_{jt}) + P_t\left[(1 - \delta_h)\overline{H}_{jt-1} + \overline{H}_{nt} - \overline{H}_{jt}\right] + 1_5^\delta(1 - \delta_h)\left[(1 - \gamma)\eta_tP_tH_{jt} + P_t(H_{jt-1} - H_{jt})\right] + (1 - 1_5^\delta)(1 - \delta_h)P_tH_{jt-1} + P_tH_{nt} - P_tH_{jt}. \quad (18)$$

The entrepreneur chooses $\{I_{jt}\}_{t=0}^{\infty}$, $\{K_{jt}\}_{t=0}^{\infty}$, $\{\overline{H}_{jt}\}_{t=0}^{\infty}$, $\{H_{jt}\}_{t=0}^{\infty}$ and $\{H_{jt}\}_{t=0}^{\infty}$ to maximize (10) subject to (8), (9), (12), (16), (17), (18), $0 \leq \overline{H}_{jt} \leq \overline{H}_{jt-1}$ and $0 \leq H_{jt} \leq H_{jt-1}$.\(^6\)

Define an entrepreneur’s value function as $V_t(\epsilon_{jt}, K_{jt-1}, \overline{H}_{jt-1}, H_{jt-1})$ where we have suppressed aggregate states. Define Tobin’s (marginal) Q as $Q_t \equiv \beta\mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \frac{\partial V_{t+1}(\epsilon_{jt+1}, K_{jt}, \overline{H}_{jt}, H_{jt})}{\partial K_{jt}}$. The

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\(^6\)The collateralized lending here can also be converted to straight sales of houses with few changes to the modeling. The insights from our model hold with either interpretation. Bigio (2015) presents an equivalence result between these two representations of modeling.
following proposition characterizes the entrepreneur’s optimal decisions on investment and house purchases.

**Proposition 2**

(i) Denote $\epsilon^*_t \equiv P_{kt}/Q_t \in (\epsilon_{\text{min}}, \epsilon_{\text{max}})$. When $\epsilon_{jt} \geq \epsilon^*_t$, entrepreneur $j$ collateralizes all his house holdings, i.e., $H'_{jt} = H_{jt-1}$ and $H'_{jt} = H_{jt-1}$, and makes real investment

$$I_{jt} = \frac{\theta_t}{P_{kt}} \left\{ P_{t\max} H_{jt-1} + \left[ 1^S_i (1 - \gamma) \eta_t P_t + (1 - 1^S_i) P_t \right] H_{jt-1} \right\}. \quad (19)$$

When $\epsilon_{jt} < \epsilon^*_t$, the entrepreneur makes no real investment, i.e., $I_{jt} = 0$. In equilibrium, all entrepreneurs are willing to hold any feasible amounts of $H_{jt}$ and $H'_{jt}$.

(ii) Tobin’s $Q$ and house prices in the equilibrium satisfy

$$Q_t = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \left[ R_{kt+1} + (1 - \delta) Q_{t+1} \right], \quad (20)$$

$$P_t = \frac{R_t}{\text{rents}} + \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \left[ \left( 1 - \delta_h \right) P_{t+1} + \theta_{t+1} \left\{ P_{t+1, \text{resale}} \left( 1^S_i \eta_{t+1} P_t + (1 - 1^S_i) P_t \right) \right\} \right], \quad (21)$$

$$P_t = \eta_t R_t + \beta (1 - \delta_h) \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \left\{ P_{t+1, \text{resale}} \left[ 1^S_i (1 - \gamma) \eta_{t+1} P_t + (1 - 1^S_i) P_t \right] \left[ 1 - F(e^*_t) \right] \right\} + \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \theta_{t+1} \left\{ \left[ 1^S_i (1 - \gamma) \eta_{t+1} P_t + (1 - 1^S_i) P_t \right] \int_{e^*_t}^{e_{max}} \left( \frac{Q_{t+1}}{P_{kt+1}} - 1 \right) dF(e) \right\}, \quad (22)$$

and usual transversality conditions.

**Proof:** Please see Appendix A.2.

Part (i) of Proposition 2 describes an entrepreneur’s decision on real investment. There exists a threshold for the idiosyncratic investment efficiency shock, $\epsilon^*_t$. When the entrepreneur receives a favorable investment efficiency shock, i.e., $\epsilon_{jt} \geq \epsilon^*_t$, he finds it profitable to invest as much as possible, so he chooses to collateralize all his houses and exhaust his borrowing limit. When he receives an unfavorable investment efficiency shock, i.e., $\epsilon_{jt} < \epsilon^*_t$, he finds it unprofitable to make any investment, so he does not borrow anything. Equation (19) demonstrates
that the investment made by an entrepreneur, if any, is limited by his collateral constraint. In
the house market, the equilibrium prices $\overline{P}_t$ and $P_t$ equate the marginal benefit of holding one
more unit of houses with the marginal cost of doing so, leaving all entrepreneurs indifferent
between purchasing and selling houses.

The two asset pricing equations of houses (21) and (22) are our key focuses. In both equa-
tions, house prices consist of three components: rents captured by terms with $R_t$, resale value
captured by terms with $1 - \delta_h$ (net of depreciation), and terms with $\theta_{t+1} \int_{\epsilon_{t+1}^*}^{\epsilon_{t+1}^{\text{max}}} \left( \frac{Q_{t+1}}{K_{t+1}} \epsilon - 1 \right) dF(\epsilon)$.
While the first two terms are common in a standard user cost model, the last term only appears
with collateral constraints. We label the last term the liquidity premium. Using equation (21) as
an example, the liquidity premium can be interpreted as follows. In period $t + 1$, when $\epsilon_{jt+1} \geq
\epsilon_{t+1}^*$, the entrepreneur collateralizes all his houses to finance real investment. Each unit of
good houses supports a loan of $\theta_{t+1} \overline{P}_{t+1}$, which generates $\theta_{t+1} \overline{P}_{t+1} \left( \frac{Q_{t+1}}{K_{t+1}} \epsilon - 1 \right)$ units of profits.
Therefore, each unit of good houses, by serving as collateral, generates $\theta_{t+1} \overline{P}_{t+1} \int_{\epsilon_{t+1}^*}^{\epsilon_{t+1}^{\text{max}}} \left( \frac{Q_{t+1}}{K_{t+1}} \epsilon - 1 \right) dF(\epsilon)$
units of expected profits. When $\epsilon_{jt+1} < \epsilon_{t+1}^*$, the entrepreneur does not invest, and the borrow-
ing constraint does not bind. Therefore, the liquidity premium reflects the option value that
houses can expand the entrepreneur’s borrowing limit whenever needed, which is also an in-
trinsic value of houses.

The liquidity premium of risky houses is interpreted analogously but depends on the lend-
ing regime. If period $t + 1$ is under II regime, then each unit of risky houses supports a loan
of $\theta_{t+1} P_{t+1}$ and generates expected profits $\theta_{t+1} P_{t+1} \left( \frac{Q_{t+1}}{K_{t+1}} \epsilon - 1 \right) dF(\epsilon)$; if period $t + 1$ is
under the IS regime, the expected profits become $(1 - \gamma) \theta_{t+1} \eta_{t+1} \overline{P}_{t+1} \int_{\epsilon_{t+1}^*}^{\epsilon_{t+1}^{\text{max}}} \left( \frac{Q_{t+1}}{K_{t+1}} \epsilon - 1 \right) dF(\epsilon)$.

With these two pricing equations, we can further prove that given the same pool of collateral,
allowing information opaqueness could sustain greater liquidity than imposing information transparency. Formally,

**Proposition 3** Assume that there exists a deterministic steady state, around which $P_t \geq (1 - \gamma) \eta_t \overline{P}_t$ with equality if $\gamma = 0$, regardless of the equilibrium regime.

**Proof:** Please see Appendix A.3.
This proposition informs us that, without acquiring the information, the market value of risky houses is always larger than the market value after acquiring the information. The underlying rationale is that, all else being equal, saving information acquisition costs is equivalent to increasing the resale value, which increases the loan size backed by the same collateral, and thus the liquidity premium. Both the resale value and liquidity premium contribute to a higher level of $P_t$. This intuition holds no matter which regime the current status and the steady state are under. When the information acquisition cost is absent, the gap between the endogenous value of risky collateral in the two regimes disappears.

This proposition reveals endogenous complementarity between the collateral quality and the lending regime. When $\eta_t$ is higher, risky houses are more likely to be good and bankers are less likely to acquire information on collateral quality. The II regime is then more likely to be implemented, which, according to Proposition 3, implies a higher market value of risky houses. This makes the condition (11) even more likely to hold and the II regime even more likely to take place than if there were no regime switching. Conversely, when $\eta_t$ is lower, bankers have stronger incentives to investigate the true quality of the houses. The information acquisition costs that are incurred in current and future periods reduce the market value of risky collateral and reinforce the occurrence of the IS regime.

### 3.2 Competitive equilibrium

Define $Y_t = \int_0^1 Y_{jt}dj$ and $K_t = \int_0^1 K_{jt}dj$ as aggregate output and capital stock, respectively. A competitive equilibrium consists of sequences of aggregate quantities \{ $C_t, I_t, N_t, Y_t, H_t, K_t$ \} and prices \{ $Q_t, W_t, R_{kt}, R_t, P_{Ht}, P_t, P_{kt}, \Lambda_t$ \} such that

(i) Households, entrepreneurs, bankers and capital producers optimize.

(ii) The markets for labor, capital and consumption goods all clear so that

\[
N_t = \int_0^1 N_{jt}dj, \quad I_t = \int_0^1 I_{jt}dj \quad \text{and} \\
Y_t = C_t + \left[ 1 + \frac{O}{2} \left( \frac{I_t}{I_{t-1}} - \exp(g) \right)^2 \right] \frac{I_t}{Z_t} + 1^S \gamma \eta_t \bar{P}_t H_t [1 - F(\epsilon_t^*)],
\]

\[ (23) \]

20
where $\gamma \eta_t H_t[1 - F(\epsilon_t^*)]$ is the total spending on information acquisition in the IS regime.

(iii) The markets for houses clear so that

$$H_t = \int_0^1 H_{jt} dj, \quad h_t = \int_0^1 H_{jt} dj \quad \text{and} \quad H_t + H_t = 1,$$

(24)

where the total supply of all houses is inelastic and normalized to 1. \(^7\)

(iv) The aggregate capital stock evolves as

$$K_t = (1 - \delta) K_{t-1} + \int_0^1 \epsilon_{jt} I_{jt} dj.$$

(25)

(v) In the II regime, good and risky houses evolve as

$$H_t = (1 - \delta_h) H_{t-1} + H_{nt}^I,$$

(26)

$$H_t = (1 - \delta_h) H_{t-1} + H_{nt}^I;$$

(27)

and in the IS regime, good and risky houses evolve as

$$H_t = (1 - \delta_h) [H_{t-1} + \eta_t H_{t-1}[1 - F(\epsilon_t^*)]] + H_{nt}^S,$$

(28)

$$H_t = (1 - \delta_h) H_{t-1} F(\epsilon_t^*).$$

(29)

We assume that $H_{nt}^I/H_{nt}^I = H_{t-1}/H_{t-1}$, i.e., the newly emerging houses have the same portfolio as the existing houses in the II regime. That is, new houses do not change the proportion of good and risky existing houses and simply compensate for the depreciated potions. In the IS regime, no risky houses emerge and $H_{nt}^S$ is pinned down by the inelastic supply of houses.

The following proposition characterizes the competitive equilibrium.

\(^7\)Saiz (2010) and Gyourko, Saiz, and Summers (2008) empirically document that the supply of houses is limited, largely due to limited supply of land. As a result, movements in house prices are dominated by movements in land prices rather than construction costs, as shown in Davis and Heathcote (2007) and Knoll, Schularick, and Steger (2017).
Proposition 4  The equilibrium system is given by equations (4), (5), (6), (14), (16), (20), (21), (22), (23), (26), (27) (for the II regime, (28) and (29) for the IS regime), $\epsilon^*_t = P_{kt}/Q_t$ and

\begin{align*}
I_t &= \frac{\theta_t}{P_{kt}} \left[ \bar{P}_t \bar{H}_{t-1} + 1^S_t (1 - \gamma) \eta_t \bar{P}_t H_{t-1} + \left( 1 - 1^S_t \right) P_t H_{t-1} \right] \left[ 1 - F(\epsilon^*_t) \right], \\
K_t &= (1 - \delta) K_{t-1} \\
&\quad + \frac{\theta_t}{P_{kt}} \left[ \bar{P}_t \bar{H}_{t-1} + 1^S_t (1 - \gamma) \eta_t \bar{P}_t H_{t-1} + \left( 1 - 1^S_t \right) P_t H_{t-1} \right] \int_{\epsilon^*_t}^{\epsilon_{\text{max}}} e dF(\epsilon), \\
N_t &= \left[ \frac{(1 - \alpha) A_t^{1-\alpha}}{W_t} \right]^{\frac{1}{\alpha}} K_{t-1}, \\
Y_t &= K_{t-1}^\alpha (A_t N_t)^{1-\alpha},
\end{align*}

for the endogenous variables \{C_t, I_t, N_t, Y_t, \bar{H}_t, H_t, K_t, W_t, Q_t, R_{kt}, R_t, \bar{P}_t, P_t, P_{kt}, \Lambda_t, \epsilon^*_t\}. The usual transversality conditions hold.

Proof. Please see Appendix B. ■

By Proposition 2, we derive aggregate investment and capital stock as in equations (30) and (31). Equation (32) is the labor market clearing condition and (33) gives aggregate output.

It is worth noting that the equilibrium path is unique after setting one regime as the default in the steady state, despite the presence of two regimes. There are no multiple equilibria given exogenous shocks and aggregate states, which allows us to solve the deterministic steady state and local dynamics around the steady state as we do for a standard real business cycle model. In the quantitative examination in Section 4, we choose the II regime as the default regime and numerically verify that there exists a unique steady state with its local dynamics on a saddle path.

4  Estimation

We detrend the equilibrium system and present it in Appendix C. We then estimate the model with Bayesian methods (An and Schorfheide, 2007). A key obstacle for doing so is that in the presence of endogenous regime switching, global solutions are computationally costly and
hinder likelihood-based estimation. We utilize the OccBin toolbox developed by Guerrieri and Iacoviello (2017), which makes the estimation accurate and efficient by estimating a piecewise linear version of a nonlinear model (Atkinson, Richter, and Throckmorton, 2020).\(^8\)

### 4.1 Parameter estimates

We categorize the parameters into two groups. The first group contains parameters that we calibrate to some stylized facts or for which we take standard values from the literature. The second group contains parameters that we estimate using quarterly data on the U.S. macroeconomy and house prices from the period 1975Q1 to 2019Q4.

**Calibration** We set the growth rate \( g = 0.005 \) to match the average annual growth rate of the real U.S. GDP per capita at 2\% according to the U.S. National Income & Product Accounts (NIPA). \( \beta \) is set equal to 0.995, implying a steady-state 2\% annual real interest rate. We set \( \kappa = 2 \), \( \alpha = 0.36 \) and \( \delta = 0.03 \), which are standard values in the real business cycle literature. We set \( \nu = 6.5 \), which lies within the empirical range in related macroeconomic and microeconomic studies (Chetty et al., 2011). We set the disutility weight on labor \( \psi_N \) to make the steady-state labor equal to 1/3. For the idiosyncratic investment efficiency shock, we assume that \( \epsilon \) follows a Pareto distribution with \( F(\epsilon) = 1 - (\epsilon_{min}/\epsilon)^\xi \), whereby we set \( \xi = 5.6 \) to match the percentage of firms making positive investments (an investment rate greater than 10\%) with its counterpart in the data (Cooper and Haltiwanger, 2006). We set \( \epsilon_{min} = (\xi - 1)/\xi \) so that \( \mathbb{E}(\epsilon) = 1 \). We choose \( \psi_H \) to replicate the imputed rental of owner-occupied nonfarm housing at 10\% of personal consumption annual expenditure, according to the NIPA. The depreciation rate of houses \( \delta_h \) is set to match an annual depreciation rate of the housing stock of 1.5\%, which is also estimated from the NIPA (Kaplan, Mitman, and Violante, 2019; Garriga, Manuelli, and Peralta-Alva, 2019). The fraction of good houses in the default regime is set at 0.2, in line with the fraction of nonsecuritized mortgage loans in all mortgage originations during the

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\(^8\)Benigno, Foerster, Otrok, and Rebucci (2020) estimate a DSGE model with an occasionally binding constraint using a second-order perturbation method.
housing boom (The Financial Crisis Inquiry Report, 2011). Here, we do not target the share of prime mortgage loans because, during the crisis, all types of securitized mortgage loans were risky, although to different degrees. For the average collateral quality $\eta$, we choose its value to simultaneously make the II regime the steady-state regime and match the average pre-crisis charge-off rate on real estate loans of all commercial banks in the U.S. (Board of Governors of the Federal Reserve System). The calibrated parameters are listed in Table 1.

[Insert Table 1 Here.]

**Estimation**  Our model contains six types of shocks: productivity shocks, housing demand shocks, labor supply shocks, collateral quality shocks, financial shocks and IST shocks. We use these shocks to match six quarterly time series of data in our estimation: real consumption per capita, real investment per capita, hours worked per capita, real house prices, the price–rent ratio and National Financial Conditions Index (NFCI). All of these observables, except the NFCI, are in logarithms, and all are demeaned.

Consumption is measured by the sum of non-housing services and non-durable goods. Investment is measured by the sum of private investment in software, equipment, structures, residential investment and expenditure on durable goods. Consumption and investment are divided by the GDP deflator. All of these data are from the U.S. NIPA. The hours worked are measured by the hours of all persons in the nonfarm business sector. To scale by population, we use the quarterly averages of the civilian non-institutional population. The latter two variables come from the Federal Reserve Economic Data at St. Louis Fed (FRED).

We use real and nominal house prices constructed by the Bank for International Settlements, based on data from Federal Reserve, CoreLogic data and National Association of Realtors. Rents are owner-equivalent imputed rents as reported by the NIPA. The price-rent ratio is calculated by nominal house prices divided by nominal rents.

To detrend these time series (except the NFCI), we use a one-sided Hodrick-Prescott (HP)
filter with a smoothing parameter at 100,000 for the pre-crisis sample. This filter is not affected by the correlation of current observations with subsequent observations and makes the detrended series suitable for estimation (Stock and Watson, 1999; Guerrieri and Iacoviello, 2017). Following the method of Bigio (2015), we apply a linear trend for the sample starting with the Great Recession. Figure 1 presents the detrended time series.

We use the NFCI as a proxy for the overall condition of U.S. financial markets. The NFCI, provided by Chicago Fed, is a weighted average of many financial activity variables and a comprehensive index of U.S. financial conditions. The NFCI’s positive values have historically been associated with tighter-than-average financial conditions, while negative values have historically been associated with looser-than-average financial conditions. To map this index into our model, we adopt the method of Miao, Wang, and Xu (2015) and define the following measurement equation:

\[
\text{NFCI}_t = -F_1 \left[ \log(\theta_t) - \log(\theta) \right] - F_2 \left[ \log(P_{ht}) - \log(P_h) \right],
\]

where the two coefficients, \( F_1 > 0 \) and \( F_2 > 0 \), are estimated. This measurement equation is motivated by the collateral constraint, which indicates that the ease of external financing varies via two dimensions: financial tightness \( \theta_t \) and collateral value \( P_{ht} \). The latter is the house price index in the model and the weighted average of prices of good and risky houses. \( P_h \) is the steady-state value of \( P_{ht} \). Either a relaxation in the overall financial condition \( \theta_t \) or an increase in the house price index \( P_{ht} \) reduces the measured NFCI.

**Priors and posteriors** The parameters that we estimate include \{\( F_1, F_2, \psi, \chi, \Omega, \theta, \gamma \}\] and those governing the stochastic processes of shocks \{\( \rho_a, \sigma_{a}, \rho_h, \sigma_h, \rho_n, \sigma_n, \rho_{\eta}, \sigma_{\eta}, \rho_\theta, \sigma_\theta, \rho_z, \sigma_z \}\]. We set the prior distributions of these estimated parameters close to those in Smets and Wouters (2007), Liu, Wang, and Zha (2013) and Miao, Wang, and Xu (2015), which cover most of the empirical estimates in the literature. For parameters that are not present in these models - specifically, the elasticity between consumption goods and housing services \( \chi \) - we set a rather
diffuse prior because its range is dispersed in the related studies.\textsuperscript{10}

We report the priors and posteriors of all the estimated parameters in Table 2. We compute the means and 5th and 95th percentiles of the posterior distributions using the Metropolis-Hastings algorithm with 200,000 draws. Our estimates of these parameters are quite robust and insensitive to the prior distributions.

[Insert Table 2 Here.]

We use the posterior modes as the parameter values for all the following results. The posterior mode of habit formation $\psi_X$ is estimated to be 0.14, close to the estimated value reported in Miao, Wang, and Zha (2020). The posterior mode of $\chi$ is 19.6. This large elasticity of substitution between consumption and housing services is mainly driven by the smooth dynamics of rents in the data. In accordance with the stylized fact in the data (Davis and Nieuwerburgh, 2015; Piazzesi and Schneider, 2016), the model-implied expenditure on housing services is fairly stable as the share of total household expenditure. The parameter for capital adjustment costs $\Omega$ is estimated to be 0.15, similar to the value estimated in Liu, Wang, and Zha (2013) and lying within the range estimated by related models (Christiano, Eichenbaum, and Evans, 2005; Miao, Wang, and Xu, 2015). The estimate for the average financial condition $\theta$ is 0.896, in line with the average loan-to-value ratio in the data (Favilukis, Ludvigson, and Nieuwerburgh, 2017; Guerrieri and Iacoviello, 2017; Garriga, Manuelli, and Peralta-Alva, 2019). The estimated value for $\gamma$ is 4.4%, which is small compared to monitoring costs used in the literature (Carlstrom and Fuerst, 1997; Christiano, Motto, and Rostagno, 2014).

Among the six shocks, collateral quality shocks have the second largest standard deviation at the posterior modes, indicating that the unconditional standard deviation of quality shocks is 4.8 times as large as that of productivity shocks. Interestingly, the estimated housing demand shocks have the smallest standard deviation, in contrast with the findings in Iacoviello (2005),

\textsuperscript{10}Flavin and Nakagawa (2008) estimate an elasticity less than 0.2. Davis and Nieuwerburgh (2015) report this elasticity around one. Based on the empirical evidence in Piazzesi, Schneider, and Tuzel (2007), this elasticity should be larger than unity. Davis and Martin (2005) show that the macro-based estimate of this elasticity is significantly larger than unity.
Liu, Wang, and Zha (2013), and Guerrieri and Iacoviello (2017). We discuss this result in more detail in Section 5.1.

4.2 Business cycle moments

We simulate the model with the calibrated and estimated parameters and compare the model-generated business cycle moments with those from the data. Table 3 summarizes these moments in the second and third columns, respectively. All of the variables are in logarithms and are HP filtered with a smoothing parameter at 1600.

[Insert Table 3 Here.]

Our model does a good job at delivering real business cycle moments compared to their counterparts in the data. In particular, the model- and data-implied house prices and the price-rent ratio are close and about two times as volatile as output. Both house prices and the price-rent ratio are positively correlated with output. These patterns are illustrated in Figure 1. The other real business cycle moments are also matched reasonably well. A drawback is the first-order autocorrelations of house prices and the price-rent ratio, which are smaller than the data. This drawback is not due to the consideration of collateral quality shocks, however, as we can see from the variants of the benchmark model in the fourth to sixth columns.

In the fourth column, we report the moments from a reference model for which we shut down the regime switching and set the II regime always as the default. While other moments do not change substantially, housing market volatilities greatly decrease, revealing the role of regime switching in exaggerating asset prices.

To see the roles of various shocks, we control for collateral quality shocks, housing demand shocks and financial shocks, one type at a time, and report the resulting moments in columns five through seven. Shutting down collateral quality shocks reduces the housing market volatilities by as much as half, whereas shutting down housing demand shocks or financial shocks has little effect on the volatilities, indicating the limited roles of these shocks in this model.
4.3 Impulse response

We now use impulse response functions to illustrate why collateral quality shocks can account for the patterns in the data. We hit the economy with a two-standard-deviation negative quality shock that is sufficiently large to trigger a regime switch from the II regime to the IS regime. Figure 2 plots the resulted impulse responses. In each panel, the solid line is the impulse response from the benchmark model, and the dashed line is from the reference model, all other things equal.

Following the negative quality shock, by (22), house prices decrease, which reduces external financing and therefore investment. It follows that output and labor decrease. At the same time, a decrease in good houses increases rents slightly, which makes the decrease of the price-rent ratio similar to that of house prices in magnitude and significantly larger than that of output. These responses are clearly consistent with the data. Because of the complementarity between consumption and housing services, consumption rises in the impact period and then falls below its steady-state value in the subsequent periods.

The responses of the benchmark model are much stronger than those of the reference model, because the regime switch gives rise to discontinuous changes and exacerbates the volatilities of the key variables. As the quality recovers, the economy returns to its original regime.

Moreover, impulse responses to collateral quality shocks embed asymmetries, as Figure 2 hints. First, when a negative shock is small, a regime switch does not occur, and the responses are just similar to the dashed lines. When a negative shock is sufficiently large, a regime switch is triggered, increasing macroeconomic volatility. Second, when a shock is positive, the condition (11) will never be violated and a regime switch will never occur. Consequently, the adverse impacts following a large negative quality shock are larger than their counterparts following an equally sized positive shock.
5 Transmission Mechanism

In this section, we aim to gain deeper insights from the model by exploring various aspects of the transmission mechanisms. We first illustrate why collateral quality shocks outperform the other relevant shocks, and then examine the relative importance of the underlying shocks with historical decomposition. We next identify the role of regime switching and finally check the model’s predictions on the financial conditions.

5.1 Comparison of shocks

Of the shocks in the model, productivity shocks, labor supply shocks and IST shocks are common in the real business cycle literature, while housing demand shocks, financial shocks and collateral quality shocks are particularly relevant to this study. We now compare the implications of the latter three shocks and illustrate why collateral quality shocks outperform the other two types of shocks in rationalizing the patterns observed in the data.

To see that, we repeat the asset pricing equation (22) below:

\[
P_t = \eta_t R_t + \beta (1 - \delta_h) \mathbb{E}_t \left\{ P_{t+1} F(\epsilon_{t+1}^*) + \left[ 1^S_{t+1} (1 - \gamma) \eta_{t+1} P_{t+1} + (1 - 1^S_{t+1}) P_{t+1} \right] \left[ 1 - F(\epsilon_{t+1}^*) \right] \right\}
\]

\[
+ \beta \mathbb{E}_t \left\{ \theta_{t+1} \left[ 1^S_{t+1} (1 - \gamma) \eta_{t+1} P_{t+1} + (1 - 1^S_{t+1}) P_{t+1} \right] \int_{\epsilon_{t+1}^*}^{\epsilon_{max}} \frac{Q_{t+1}^*}{P_{kt+1}^*} dF(\epsilon) \right\}
\]

liquidity premium

From a theoretical point of view, the standard approach to pricing houses treats houses as an asset that delivers rents as dividends. House prices are then simply the sum of present values of future rents, which are the marginal utility of housing (Favilukis, Ludvigson, and Nieuwerburgh, 2017). This is the “house rent channel.” Here, the above equation highlights another channel associated with collateral constraints, which we refer to as the “liquidity premium channel.” On the one hand, if we only consider shifts in housing demand, then there arises a puzzle that house prices and rents increase or decrease at roughly the same pace, which is counterfactual to the stylized fact that house prices are significantly more volatile than rents.
On the other hand, the above equation tells us that collateral quality shocks affect house prices more than rents by affecting (i) the probability that houses are good or toxic, (ii) the liquidity premium and (iii) switching between the two lending regimes. As a result, collateral quality shocks amplify fluctuations in the price-rent ratio, as shown in Figure 2, and therefore helps resolve the puzzle.

Through the lens of these two channels, we now analyze the implications of housing demand shocks and financial shocks, both of which have received much attention in the literature.

**Housing demand shock** Figure 3 plots impulse responses to a negative one-standard-deviation housing demand shock. When housing demand decreases, rents decrease, causing house prices to decrease. The housing demand shock mainly works through the house rent channel and only indirectly affects the liquidity premium. Consequently, the decrease in house prices is smaller than the decrease in rents, implying a counterfactual increase in the price-rent ratio. In other words, housing demand shocks cannot generate a positive correlation between house prices and the price-rent ratio.

[Insert Figure 3 Here.]

The above comparison justifies why collateral quality shocks are estimated to be large while housing demand shocks are estimated to be small, which seemingly contrasts with Iacoviello (2005), Liu, Wang, and Zha (2013) and Guerrieri and Iacoviello (2017). However, this difference is not surprising because the estimations in these studies do not include the data for the price-rent ratio. As pointed out by Liu, Wang, and Zha (2019), it is important to consider the role of the liquidity premium to simultaneously account for house prices and the price-rent ratio.

**Financial shock** Figure 4 displays impulse responses to a negative one-standard-deviation financial shock. Given the collateral value, a negative financial shock tightens credit constraints and depresses investment and output. However, with the tightened financial conditions, the
demand for collateral increases, pushing up the price of collateral. As a result, house prices and
the price-rent ratio increase, implying that house prices are countercyclical. This counterfac-
tual implication casts doubt on a dominant role of financial shocks in generating the observed
patterns.

[Insert Figure 4 Here.]

This result echoes a number of recent studies, such as Kiyotaki, Michaelides, and Nikolov
Kaplan, Mitman, and Violante (2019) and Justiniano, Primiceri, and Tambalotti (2019). With a
variety of models, these studies all find changes in credit conditions alone have limited effects
on house prices and rents.

**Joint shocks** As analyzed above, neither housing demand shocks nor financial shocks alone
are likely to dominate the dynamics of output, house prices, and the price-rent ratio. We now
explain why these two types of shocks can hardly explain the patterns jointly. To mitigate their
counterfactual implications, the sizes of these two types of shocks must be jointly determined
by the data. However, because house prices and the price-rent ratio track each other closely in
the data (see Figure 1), which is strongly against the implication of housing demand shocks,
housing demand shocks must be small. It immediately follows that financial shocks cannot be
significant or that housing demand shocks are insufficient to reverse the counterfactual impli-
cation of financial shocks. For this reason, the dynamics in the data are unlikely to be attributed
to housing demand shocks, financial shocks, or a combination of these two.

### 5.2 Historical decomposition

In this section, we assess the relative importance of the various shocks in accounting for the his-
torical path of the macroeconomy, and especially during the Great Recession. Figure 5 presents
the decomposition of house prices, the price-rent ratio and investment in the estimated model.
The shocks are marginalized in the following order: (i) collateral quality shock, (ii) productivity shock and financial shock, (iii) housing demand shock, (iv) labor supply shock, and (v) IST shock. The height of a single-color column denotes the marginal contribution of the corresponding shock(s) to a variable in a period, with the marginal contribution of all shocks totaling to the observed time series.

The decomposition shows that collateral quality shocks explain the largest share of the movements in house prices and the price-rent ratio, and shows clear deterioration in collateral quality at the turning point from the boom to the bust. During the 2003-2006 housing boom, collateral quality shocks account for 50% to 60% of the variations in house prices and the price-rent ratio caused by all of the shocks and 20% to 25% of the variations in investment. In the Great Recession, taking 2009Q2 as an example, collateral quality shocks account for 19% out of the 28% decline in house prices, 21% out of the 29% decline in the price-rent ratio, and 6% out of the 29% decline in investment. Overall, collateral quality shocks contribute more than half of the variations and approximately one-fifth of the variations in investment during the boom and bust.

Productivity shocks also matter for the crisis, consistent with the quantitative findings in Bigio (2015) and Favilukis, Ludvigson, and Nieuwerburgh (2017). The rest of the variations in investment are attributed to IST shocks and labor supply shocks. The contribution of housing demand shocks and financial shocks is minor, echoing our discussion in the previous section.

5.3 The role of regime switching

To evaluate the role of regime switching, we estimate the reference model, in which we eliminate the regime switching. The benchmark and reference models share the same calibrated parameter values and the same prior distributions for the same set of estimated parameters, so these two models are viewed as equally likely a priori. We report the fitness of these two models in Table 4 and the estimated parameters of the reference model in Table 5. The log
marginal densities of the data for the benchmark and reference models are 3012.8 and 2983.3, respectively, indicating that the data favors the benchmark model over the reference model.

[Insert Tables 4 and 5 Here.]

Figure 2 offers support for the superior performance of the benchmark model. Without regime switching, the reference model needs larger shocks, reflected by stronger persistence and larger standard deviations, to fit the declines observed during the Great Recession, resulting in a lower marginal density of this model relative to the benchmark model.

To see the endogenous regime switch in the estimated sample identified by our model, we feed the estimated sequences of shocks into these two models. Figure 6 plots the paths in which the solid lines, dashed lines and “plus” signs denote the benchmark model, reference model and data, respectively.

[Insert Figure 6 Here.]

In all of the panels, the fitted series from the benchmark model tracks the actual series as designed. More importantly, the series from the benchmark and reference model coincide with each other until the beginning of the Great Recession and diverge from each other thereafter. Such divergence marks an endogenous regime switch and demonstrates that if the regime switch had not occurred, house prices would have not decreased as much as they did. Similarly, without the regime switch, the price-rent ratio, investment and consumption would have been greater.

The gaps between the solid and dashed lines measure the contribution of the regime switch per se to movements in these variables. In 2014Q3, when the U.S. ended QE3, In particular, the regime switch caused additional drops of 3.5% in investment, 1.2% in consumption, 12.4% in house prices, and 12.3% in the price-rent ratio. These gaps are non-negligible compared to the decreases in these variables throughout the whole recession (2007Q4 to 2009Q2): 29%, 5%, 27% and 27%, respectively. In summary, the regime switch create adverse impacts on the macroeconomy.
5.4 Model prediction on financial market

Finally, we examine whether our estimated model delivers a reasonable historical path of financial tightness. In the model, this is represented by financial shocks $\theta_t$ which are orthogonal to collateral quality shocks. For the proxy of financial tightness in the data, we do not use the NFCI because (i) the NFCI covers not only the indexes for financial tightness but also indexes on house prices and (ii) it is already matched in the estimation. Instead, we use the net percentage of domestic banks tightening standards for commercial real estate loans, released by the Senior Loan Officer Survey from the U.S. Board of Governors of the Federal Reserve System. This is a direct measurement of financial tightness and not intentionally targeted in the estimation. As a tightened condition with this proxy is mapped to a low value of $\theta_t$, the model- and data-implied financial tightness are expected to move in opposite directions.

[Insert Figure 7 Here.]

As displayed in Figure 7, the model replicates the dynamics of financial tightness in the data well. The credit condition was loose before and after the crisis and was squeezed during the crisis. This additional performance test corroborates the validity of our model in explaining the credit boom and bust.

6 Conclusion

The key friction that we consider in this paper is that collateral quality could be imperfect and the true quality of collateral is not freely observable or assessable by most participants in financial markets. Focusing on real estate collateral, we find that shifts in the collateral quality can affect house prices more strongly than rents, which helps resolve the puzzle of the high volatility of the price-rent ratio. Relatedly, there also arise two lending regimes depending on whether lenders are induced to pay an information acquisition cost and learn the exact quality of risky collateral. Thus, the model features endogenous regime switching accompanying shifts in collateral quality, which adds volatilities to asset prices and the macroeconomy.
According to our estimation of the model with Bayesian methods, we find that collateral quality shocks account for more than a half of the variations in house prices and the price-rent ratio and account for approximately a fifth of the variations in investment during the housing boom and bust. Moreover, the model can identify an endogenous regime switch associated with the crisis, and the data favors the benchmark model over the reference model for which we shut down the regime switching. In summary, our study underscores the importance of collateral quality shocks and associated information friction in explaining the prominent patterns in housing markets.
References


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A Appendix

A.1 Proof of Lemma 2

By condition (11),

\[ \frac{P_t}{\bar{P}_t} > \frac{\eta_t}{\gamma + \eta_t} > \frac{\eta_t}{\gamma + 1} > (1 - \gamma) \eta_t. \]

Q.E.D.

A.2 Proof of Proposition 2

We can write the dynamic programming of an entrepreneur \( j \) as follows

\[
V_t(\epsilon_{jt}, K_{jt-1}, H_{jt-1}, H_{jt-1}) = \max \left\{ D_{jt} + \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1}(\epsilon_{jt}, K_{jt}, H_{jt}, H_{jt}), \right. \\
\left. \quad \left\{ I_{jt}, K_{jt}, H'_{jt}, H_{jt} \right\} \right\}
\]

subject to (8), (9), (12), (16), (17), (18), 0 \( \leq H'_{jt} \leq H_{jt-1} \) and 0 \( \leq H'_{jt} \leq H_{jt-1} \).

We conjecture that the above value function takes the following form

\[
V_t(\epsilon_{jt}, K_{jt-1}, H_{jt-1}, H_{jt-1}) = \Phi_{Kt}(\epsilon_{jt}) K_{jt-1} + \Phi_{Ht}(\epsilon_{jt}) H_{jt-1} + \Phi_t
\]

where \( \Phi_{Kt}(\epsilon_{jt}), \Phi_{Ht}(\epsilon_{jt}) \) and \( \Phi_t \) are coefficients to be determined.

By the definition, Tobin’s Q satisfies

\[
Q_t \equiv \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \frac{\partial V_{t+1}(\epsilon_{jt+1}, K_{jt}, H_{jt}, H_{jt})}{\partial K_{jt}} = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \int_{\epsilon_{\min}}^{\epsilon_{\max}} \Phi_{Kt+1}(\epsilon) dF(\epsilon).
\]
We also conjecture that

\[
\begin{align*}
\mathcal{P}_t &= R_t + \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \int_{\epsilon_{\min}}^{\epsilon_{\max}} \Phi_{H_{t+1}}(\epsilon) d\mathcal{F}(\epsilon), \\
P_t &= \eta_t R_t + \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \int_{\epsilon_{\min}}^{\epsilon_{\max}} \Phi_{H_{t+1}}(\epsilon) d\mathcal{F}(\epsilon).
\end{align*}
\]

Substituting (8), (18), (36), (37) and (38) into the right-hand side of the Bellman equation (34), we obtain

\[
D_{jt} + \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} V_{t+1}(\epsilon_{jt+1}, K_{jt}, \bar{H}_{jt}, H_{jt})
\]

\[
= R_{kt} K_{jt-1} - P_{kt} I_{jt} + R_t (\bar{H}_{jt} + \eta_t H_{jt}) + \bar{P}_t [(1 - \delta_h) \bar{H}_{jt-1} - \bar{H}_{jt}]
\]

\[
+ \left\{1^S_t \left[ (1 - \gamma) \eta_t \bar{P}_t H'_{jt} + P_t (H_{jt-1} - H'_{jt}) \right] + \left(1 - 1^S_t\right) (1 - \delta_h) P_t H_{jt-1} - P_t H_{jt}\right\}
\]

\[+(1 - \delta) Q_t K_{jt-1} + \epsilon_{jt} Q_t I_{jt} + (\bar{P}_t - R_t) \bar{H}_{jt} + (P_t - \eta_t R_t) H_{jt} + \Phi_t
\]

\[
= R_{kt} K_{jt-1} + (1 - \delta_h) \bar{P}_t \bar{H}_{jt-1} + 1^S_t (1 - \delta_h) \left[ (1 - \gamma) \eta_t \bar{P}_t H'_{jt-1} + P_t (H_{jt-1} - H'_{jt-1}) \right]
\]

\[+(1 - 1^S_t) (1 - \delta_h) P_t H_{jt-1} + (1 - \delta) Q_t K_{jt-1} + (\epsilon_{jt} Q_t - P_{kt}) I_{jt} + \Phi_t,
\]

where \(\Phi_t\) absorbs terms containing \(\bar{H}_{nt}\) and \(H_{nt}\).

When \(\epsilon_{jt} < P_{kt}/Q_t\), the entrepreneur finds it unprofitable to make any investment, so he does not borrow anything, i.e., \(I_{jt} = H'_{jt} = H'_{jt-1} = 0\). When \(\epsilon_{jt} \geq P_{kt}/Q_t\), the entrepreneur finds it profitable to make investment as much as possible, so he exhausts his borrowing limit, i.e.,

\[
\bar{H}_{jt-1} = \bar{H}_{jt-1}, \quad H'_{jt-1} = H_{jt-1},
\]

\[
P_{kt} I_{jt} = \theta_t \left[ \bar{P}_t \bar{H}_{jt-1} + 1^S_t (1 - \gamma) \eta_t \bar{P}_t H_{jt-1} + \left(1 - 1^S_t\right) P_t H_{jt-1}\right],
\]

where the investment \(I_{jt}\) is pinned down by (9) and (12). We then obtain (19). At the equilibrium house prices \(\bar{P}_t\) and \(P_t\), the entrepreneur is indifferent between purchasing and selling both types of houses.
Substituting the above decisions into the Bellman equation, we obtain

\[
V_t(\varepsilon_{jt}, K_{jt-1}, H_{jt-1}) = \begin{cases} 
R_{kt}K_{jt-1} + (1 - \delta)Q_tK_{jt-1} + (1 - \delta_h)\bar{P}_tH_{jt-1} + \Phi_t \\
+ \mathbf{1}_i^S(1 - \delta_h)(1 - \gamma)\eta_t\bar{P}_tH_{jt-1} + (1 - \mathbf{1}_i^S)(1 - \delta_h)P_tH_{jt-1} \\
+ \theta_t \left( P_{kt}^*\varepsilon_{jt} - 1 \right) \left[ \bar{P}_tH_{jt-1} + \mathbf{1}_i^S(1 - \gamma)\eta_t\bar{P}_tH_{jt-1} + (1 - \mathbf{1}_i^S)P_tH_{jt-1} \right], & \text{if } \varepsilon_{jt} \geq \varepsilon_t^*; \\
R_{kt}K_{jt-1} + (1 - \delta)Q_tK_{jt-1} + (1 - \delta_h)\bar{P}_tH_{jt-1} + (1 - \delta_h)P_tH_{jt-1} + \Phi_t, & \text{if } \varepsilon_{jt} < \varepsilon_t^*. 
\end{cases}
\]

Matching coefficients \(\Phi_{Kt}(\varepsilon_{jt})\), \(\Phi_{Pt}(\varepsilon_{jt})\) and \(\Phi_{Ht}(\varepsilon_{jt})\) in the above equation and (35), and making use of equations (36), (37) and (38) yield (20), (21) and (22). Q.E.D.

### A.3 Proof of Proposition 3

By Lemma 2 and equation (22), we obtain

\[
P_t \geq \eta_t R_t + \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \left\{ (1 - \delta_h)(1 - \gamma)\eta_{t+1}\bar{P}_{t+1}F(\varepsilon_{t+1}^*) \\
+ (1 - \delta_h) \left[ \mathbf{1}_i^{S_{t+1}}(1 - \gamma)\eta_{t+1}\bar{P}_{t+1} + (1 - \mathbf{1}_i^{S_{t+1}})(1 - \gamma)\eta_{t+1}\bar{P}_{t+1} \right] \left[ 1 - F(\varepsilon_{t+1}^*) \right] \\
+ \theta_{t+1} \left[ \mathbf{1}_i^{S_{t+1}}(1 - \gamma)\eta_{t+1}\bar{P}_{t+1} + (1 - \mathbf{1}_i^{S_{t+1}})(1 - \gamma)\eta_{t+1}\bar{P}_{t+1} \right] \int_{\varepsilon_{t+1}^*}^{\varepsilon_{\max}} \left( \frac{Q_{t+1}}{P_{kt+1}} - 1 \right) dF(\varepsilon) \right\} \\
= \eta_t R_t + \beta (1 - \gamma) \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \eta_{t+1}\bar{P}_{t+1} \left[ 1 - \delta_h + \theta_{t+1} \int_{\varepsilon_{t+1}^*}^{\varepsilon_{\max}} \left( \frac{Q_{t+1}}{P_{kt+1}} - 1 \right) dF(\varepsilon) \right].
\]

(39)

Detrending the above inequality (the detrending rule is given in Appendix C) leads to

\[
p_t \geq \eta_t r_t + \beta (1 - \gamma) \exp[(1 - \kappa)g] \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \eta_{t+1}\bar{P}_{t+1} \left[ 1 - \delta_h + \theta_{t+1} \int_{\varepsilon_{t+1}^*}^{\varepsilon_{\max}} \left( \frac{Q_{t+1}}{P_{kt+1}} - 1 \right) dF(\varepsilon) \right].
\]

(40)

Letting \(\hat{m}_t\) denote the percentage deviation of a variable \(m_t\) around its steady-state value,
the log-linearized version of the above inequality around the steady state is

\[
\hat{p}_t \geq \Gamma (\hat{\eta}_t + \hat{r}_t) + (1 - \Gamma) E_t \hat{\eta}_{t+1} + (1 - \Gamma) \left\{ E_t \frac{\lambda_{t+1}}{\lambda_t} \hat{p}_{t+1} \left[ 1 - \delta_h + \theta_{t+1} \int_{\epsilon^{\text{max}}_{t+1}} (\frac{Q_{t+1}}{P_{kt+1}} e - 1) dF(\epsilon) \right] \right\}
\]

\[
= \hat{\eta}_t + \Gamma \hat{r}_t + (1 - \Gamma) \left\{ E_t \frac{\lambda_{t+1}}{\lambda_t} \hat{p}_{t+1} \left[ 1 - \delta_h + \theta_{t+1} \int_{\epsilon^{\text{max}}_{t+1}} (\frac{Q_{t+1}}{P_{kt+1}} e - 1) dF(\epsilon) \right] \right\}, \tag{41}
\]

where we utilize \( \hat{\eta}_t = E_t \hat{\eta}_{t+1} \) implied by the stochastic process of \( \eta_t \) and define

\[
\Gamma = \frac{r}{r + \beta (1 - \gamma) \exp[(1 - \kappa)g] \overline{p} \left[ 1 - \delta_h + \theta \int_{\epsilon^{\text{max}}} (\frac{Q}{P_k} e - 1) dF(\epsilon) \right]}. \tag{42}
\]

Equation (21) is detrended as

\[
\overline{p}_t = r + \beta \exp[(1 - \kappa)g] E_t \frac{\lambda_{t+1}}{\lambda_t} \overline{p}_{t+1} \left[ 1 - \delta_h + \theta \int_{\epsilon^{\text{max}}} (\frac{Q_{t+1}}{P_{kt+1}} e - 1) dF(\epsilon) \right]. \tag{43}
\]

Log-linearizing it around the steady state yields

\[
\hat{p}_t = \Gamma \hat{r}_t + (1 - \Gamma) \left\{ E_t \frac{\lambda_{t+1}}{\lambda_t} \overline{p}_{t+1} \left[ 1 - \delta_h + \theta_{t+1} \int_{\epsilon^{\text{max}}_{t+1}} (\frac{Q_{t+1}}{P_{kt+1}} e - 1) dF(\epsilon) \right] \right\}. \tag{44}
\]

Combining (41) and (44), we have

\[
\hat{p}_t \geq \hat{\eta}_t + \overline{p}_t. \tag{45}
\]

Next we show that \( p \geq (1 - \gamma) \eta \overline{p} \) in the deterministic steady state. If the steady-state regime is the II regime, then the steady-state versions of (21) and (22) become

\[
\overline{p} = r + \beta \exp[(1 - \kappa)g] \overline{p} \left[ 1 - \delta_h + \theta \int_{\epsilon^{\text{max}}} (\frac{Q}{P_k} e - 1) dF(\epsilon) \right], \tag{46}
\]

\[
p = \eta r + \beta \exp[(1 - \kappa)g] p \left[ 1 - \delta_h + \theta \int_{\epsilon^{\text{max}}} (\frac{Q}{P_k} e - 1) dF(\epsilon) \right], \tag{47}
\]

comparing which yields \( p = \eta \overline{p} \).
If the steady-state regime is the IS regime, then the steady-state version of (40) becomes

\[ p \geq \eta r + \beta (1 - \gamma) \exp[(1 - \kappa)g] \eta \bar{p} \left[ 1 - \delta_h + \theta \int_{\epsilon^*}^{\epsilon_{\max}} \left( \frac{Q}{P_k} \epsilon - 1 \right) dF(\epsilon) \right] \]

\[ = (1 - \gamma) \eta r + \beta (1 - \gamma) \exp[(1 - \kappa)g] \eta \bar{p} \left[ 1 - \delta_h + \theta \int_{\epsilon^*}^{\epsilon_{\max}} \left( \frac{Q}{P_k} \epsilon - 1 \right) dF(\epsilon) \right] \]

\[ = (1 - \gamma) \eta \bar{p}, \tag{48} \]

where the last equality is due to (46).

Combining (45) and (48) generates \( p_t \geq (1 - \gamma) \eta_t \bar{p}_t \) or \( P_t \geq (1 - \gamma) \eta_t \bar{p}_t \). When \( \gamma = 0 \), all inequality above become equality and then \( P_t = \eta_t \bar{p}_t \). Q.E.D.

B Equilibrium System

Here we derive equations (30) to (33). The other equations in the equilibrium system are derived before. We use the decision rule in Proposition 2 and the Law of Large Numbers to derive aggregate investment,

\[ I_t = \int_0^1 I_{jt} dj \]

\[ = \int_0^1 \mathcal{1}(\epsilon_{jt} \geq \epsilon_{jt}^*) \frac{\theta_t}{P_{kt}} \left[ \bar{p}_t H_{jt-1} + \mathcal{1}_t S(t - \gamma) \eta_t \bar{p}_t H_{jt-1} + \left( 1 - \mathcal{1}_t \right) \bar{p}_t H_{jt-1} \right] dj + \int_0^1 \mathcal{1}(\epsilon_{jt} < \epsilon_{jt}^*) dj \]

\[ = \frac{\theta_t}{P_{kt}} \left[ \bar{p}_t H_{jt-1} + \mathcal{1}_t S(t - \gamma) \eta_t \bar{p}_t H_{jt-1} + \left( 1 - \mathcal{1}_t \right) \bar{p}_t H_{jt-1} \right] \left[ 1 - F(\epsilon_{jt}^*) \right], \]

where the last equality is due to the fact that \( \epsilon_{jt} \) is IID across entrepreneurs. We obtain (30).

Similarly, we derive the evolution of aggregate capital stock as

\[ K_t = (1 - \delta) K_{t-1} + \int_0^1 \epsilon_{jt} I_{jt} dj \]

\[ = (1 - \delta) K_{t-1} + \frac{\theta_t}{P_{kt}} \left[ \bar{p}_t H_{jt-1} + \mathcal{1}_t S(t - \gamma) \eta_t \bar{p}_t H_{jt-1} + \left( 1 - \mathcal{1}_t \right) \bar{p}_t H_{jt-1} \right] \int_{\epsilon_t^*}^{\epsilon_{\max}} \epsilon dF(\epsilon), \]

which is (31).
The entrepreneur’s labor demand problem (15) gives

\[ N_{jt} = \left[ \frac{(1 - \alpha)A_t^{1-\alpha}}{W_t} \right]^{\frac{1}{\alpha}} K_{jt-1}. \] (49)

The labor market clearing condition implies that

\[ N_t = \int_0^1 N_{jt}dj = \left[ \frac{(1 - \alpha)A_t^{1-\alpha}}{W_t} \right]^{\frac{1}{\alpha}} K_{t-1}, \] (50)

which is (32).

Substituting (49) into production function (7), we derive aggregate output

\[ Y_t = \int_0^1 Y_{jt}dj = \int_0^1 K_{jt-1}^{\frac{\alpha}{1-\alpha}} \left[ \frac{(1 - \alpha)A_t^{1-\alpha}}{W_t} \right]^{\frac{1}{\alpha}} K_{jt-1}^{1-\alpha} dj = K_t^{\frac{\alpha}{1-\alpha}} (A_t N_t)^{1-\alpha}, \]

which is (33).

Substituting the flow-of-funds constraints of entrepreneurs, bankers and capital producers,

\[ D_e^t = \int_0^1 D_{jt} dj, \]

\[ D_b^t = 0 \]

\[ D_k^t = P_{kt} I_t - \left[ 1 + \Omega \left( \frac{I_t}{I_{t-1}} - \exp(g) \right)^2 \right] \frac{I_t}{Z_t} \]

into (3), we obtain

\[ C_t + \left[ 1 + \Omega \left( \frac{I_t}{I_{t-1}} - \exp(g) \right)^2 \right] \frac{I_t}{Z_t} + \gamma H_t [1 - \mathcal{F}(\epsilon_t^*)] = Y_t, \]

which is (23). Q.E.D.

C Detrended Equilibrium System

We can verify that the equilibrium variables \( \epsilon_t^*, Q_t, R_{kt}, P_{kt}, N_t, \bar{H}_t \) and \( H_t \) do not have trend. All the other equilibrium variables in Proposition 4 grow around the balanced growth path at the rate \( g \) except for \( \Lambda_t \). Letting \( \Lambda_t = \lambda_t \exp(-\kappa g t) \) and any other growing variable \( M_t = \)
\[ m_t \exp(gt), \text{ we detrend all the conditions in Proposition 4 and obtain the following system:} \]

\[
\begin{align*}
\lambda_t &= (1 - \psi_{Ht})x_t^\frac{1}{\beta} - \frac{1}{\lambda} \left[ \left( x_t - \psi x \frac{x_{t-1}}{\exp(g)} - \psi_{Nt} \frac{N_{t+1}^1 + \nu}{1 + \nu} \right)^{-\kappa} \right] - \beta \psi x \exp(-\kappa g) \mathbb{E}_t \left( x_{t+1} - \psi x \frac{x_t}{\exp(g)} - \psi_{Nt+1} \frac{N_{t+1}^1 + \nu}{1 + \nu} \right)^{-\kappa}, \\
\omega_t \lambda_t &= \left( x_t - \psi x \frac{x_{t-1}}{\exp(g)} - \psi_{Nt} \frac{N_{t+1}^1 + \nu}{1 + \nu} \right)^{-\kappa} \psi_{Nt} N_t^\nu, \\
(1 - \psi_{Ht})r_t c_t^{-\frac{1}{\beta}} &= \psi_{Ht} \left( \overline{H}_t + \eta_t H_t \right)^{-\frac{1}{\beta}}, \\
Z_t p_k t &= 1 + \frac{\Omega}{2} \exp(2g) \left( \frac{i_t}{i_{t-1}} - 1 \right)^2 + \Omega \exp(2g) \left( \frac{i_t}{i_{t-1}} - 1 \right) \frac{i_t}{i_{t-1}} \\
&- \beta \exp([3 - \kappa] g) \mathbb{E}_t \left( \frac{i_{t+1}}{i_t} - 1 \right) \frac{Z_t}{Z_{t+1}} \left( \frac{i_{t+1}}{i_t} \right)^2, \\
R_{kt} &= \alpha \left[ (1 - \alpha) a_t \right]^{\frac{1}{\alpha}} w_t, \\
y_t &= c_t + \left[ 1 + \frac{\Omega}{2} \exp(2g) \left( \frac{i_t}{i_{t-1}} - 1 \right)^2 \right] \frac{i_t}{Z_t} + 1_t^S \gamma_t \eta_t \overline{p}_t H_t \left[ 1 - \mathcal{F}(\epsilon_t^*) \right], \\
Q_t &= \beta \exp(-\kappa g) \mathbb{E}_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( \frac{i_{t+1}}{i_t} - 1 \right) \frac{Z_t}{Z_{t+1}} \left( \frac{i_{t+1}}{i_t} \right)^2 \right] \left[ 1 - \delta_{t+1} + \theta_{t+1} \int_{\epsilon_{t+1}^*)}^{\epsilon_{\max}} \left( \frac{Q_{t+1}}{P_{kt+1}} \epsilon - 1 \right) d\mathcal{F}(\epsilon) \right], \\
p_t &= \eta_t r_t + \beta \exp([1 - \kappa] g) \mathbb{E}_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \right] \left( 1 - \delta_{t+1} \right) p_{t+1} \mathcal{F}(\epsilon_{t+1}^*) \\
&+ (1 - \delta_{t+1}) \left[ 1_t^S (1 - \gamma) \eta_t \overline{p}_{t+1} + (1 - 1_t^S) p_{t+1} \right] \left[ 1 - \mathcal{F}(\epsilon_{t+1}^*) \right] \\
&+ \theta_{t+1} \left[ 1_t^S (1 - \gamma) \eta_t \overline{p}_{t+1} + (1 - 1_t^S) p_{t+1} \right] \int_{\epsilon_{t+1}^*)}^{\epsilon_{\max}} \left( \frac{Q_{t+1}}{P_{kt+1}} \epsilon - 1 \right) d\mathcal{F}(\epsilon), \\
\epsilon_t^* &= \frac{P_{kt}}{Q_t}, \\
i_t &= \frac{\theta_t}{P_{kt}} \left[ \overline{p}_t \overline{H}_{t-1} + 1_t^S (1 - \gamma) \eta_t \overline{p}_t H_{t-1} + (1 - 1_t^S) p_t H_{t-1} \right] \left[ 1 - \mathcal{F}(\epsilon_t^*) \right], \\
k_t &= (1 - \delta) \frac{k_{t-1}}{\exp(g)} \\
&+ \frac{\theta_t}{P_{kt}} \left[ \overline{p}_t \overline{H}_{t-1} + 1_t^S (1 - \gamma) \eta_t \overline{p}_t H_{t-1} + (1 - 1_t^S) p_t H_{t-1} \right] \int_{\epsilon_t^*)}^{\epsilon_{\max}} \epsilon d\mathcal{F}(\epsilon), \quad (63)
\end{align*}
\]
\[
N_t = \left[ \frac{(1-\alpha)a_t^{1-\alpha}}{w_t} \right]^\frac{1}{\alpha} k_{t-1} \frac{\exp(g)}{\exp(g)} \\
y_t = \left[ \frac{k_{t-1}}{\exp(g)} \right]^\alpha (a_t N_t)^{1-\alpha},
\]

(64) and (65) for the II regime, and (28) and (29) for the IS regime.
### Table 1: Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>0.005</td>
<td>Average quarterly growth rate of aggregate productivity</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.995</td>
<td>Discount factor</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>2</td>
<td>Intertemporal elasticity of substitution</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.36</td>
<td>Capital share in production</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.03</td>
<td>Capital depreciation rate</td>
</tr>
<tr>
<td>$\delta_h$</td>
<td>0.004</td>
<td>House depreciation rate</td>
</tr>
<tr>
<td>$\psi_H$</td>
<td>0.1</td>
<td>Utility weight on housing services</td>
</tr>
<tr>
<td>$\nu$</td>
<td>6.5</td>
<td>Inverse of Frisch elasticity</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>5.6</td>
<td>Shape of the distribution of idiosyncratic investment efficiency</td>
</tr>
<tr>
<td>$\overline{H}$</td>
<td>0.2</td>
<td>Fraction of good houses</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.974</td>
<td>Average collateral quality</td>
</tr>
</tbody>
</table>
Table 2: Estimated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior Distribution</th>
<th>Posterior Distribution</th>
<th>5%</th>
<th>95%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Distr.</td>
<td>Mean</td>
<td>S.D.</td>
<td>Mode</td>
</tr>
<tr>
<td>$F_1$</td>
<td>Gamma</td>
<td>2</td>
<td>2</td>
<td>1.655</td>
</tr>
<tr>
<td>$F_2$</td>
<td>Gamma</td>
<td>2</td>
<td>2</td>
<td>0.129</td>
</tr>
<tr>
<td>$\psi_X$</td>
<td>Beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.138</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Gamma</td>
<td>10</td>
<td>5</td>
<td>19.644</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Gamma</td>
<td>2</td>
<td>2</td>
<td>0.150</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.896</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Gamma</td>
<td>0.05</td>
<td>0.02</td>
<td>0.044</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>Beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.967</td>
</tr>
<tr>
<td>$\rho_h$</td>
<td>Beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.974</td>
</tr>
<tr>
<td>$\rho_n$</td>
<td>Beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.945</td>
</tr>
<tr>
<td>$\rho_\eta$</td>
<td>Beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.986</td>
</tr>
<tr>
<td>$\rho_\theta$</td>
<td>Beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.988</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>Beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.961</td>
</tr>
<tr>
<td>$\sigma_a$ (%)</td>
<td>Inv. Gamma</td>
<td>1</td>
<td>Inf</td>
<td>1.281</td>
</tr>
<tr>
<td>$\sigma_h$ (%)</td>
<td>Inv. Gamma</td>
<td>1</td>
<td>Inf</td>
<td>0.625</td>
</tr>
<tr>
<td>$\sigma_n$ (%)</td>
<td>Inv. Gamma</td>
<td>1</td>
<td>Inf</td>
<td>7.448</td>
</tr>
<tr>
<td>$\sigma_\eta$ (%)</td>
<td>Inv. Gamma</td>
<td>1</td>
<td>Inf</td>
<td>4.282</td>
</tr>
<tr>
<td>$\sigma_\theta$ (%)</td>
<td>Inv. Gamma</td>
<td>1</td>
<td>Inf</td>
<td>2.151</td>
</tr>
<tr>
<td>$\sigma_z$ (%)</td>
<td>Inv. Gamma</td>
<td>1</td>
<td>Inf</td>
<td>0.880</td>
</tr>
</tbody>
</table>

Note: The posterior distribution is obtained using the Metropolis-Hastings algorithm with 200,000 draws.
Table 3: Real Business Cycle Moments

<table>
<thead>
<tr>
<th>Moment Data</th>
<th>Benchmark</th>
<th>No regime switching</th>
<th>No quality shock</th>
<th>No housing demand shock</th>
<th>No financial shock</th>
</tr>
</thead>
<tbody>
<tr>
<td>std(y_t) (%)</td>
<td>1.75</td>
<td>1.47</td>
<td>1.46</td>
<td>1.46</td>
<td>1.47</td>
</tr>
<tr>
<td>std(c_t) / std(y_t)</td>
<td>0.49</td>
<td>0.69</td>
<td>0.69</td>
<td>0.68</td>
<td>0.69</td>
</tr>
<tr>
<td>std(i_t) / std(y_t)</td>
<td>2.25</td>
<td>2.49</td>
<td>2.46</td>
<td>2.44</td>
<td>2.48</td>
</tr>
<tr>
<td>std(n_t) / std(y_t)</td>
<td>0.99</td>
<td>0.96</td>
<td>0.96</td>
<td>0.96</td>
<td>0.96</td>
</tr>
<tr>
<td>std(p_{ht}) / std(y_t)</td>
<td>1.78</td>
<td>1.84</td>
<td>1.37</td>
<td>0.88</td>
<td>1.83</td>
</tr>
<tr>
<td>std(p_{ht} / r_t) / std(y_t)</td>
<td>1.86</td>
<td>1.98</td>
<td>1.51</td>
<td>0.92</td>
<td>1.94</td>
</tr>
</tbody>
</table>

Correlation with output

| corr(c_t, y_t) | 0.83      | 0.96                | 0.96             | 0.97                   | 0.96             |
| corr(i_t, y_t) | 0.97      | 0.83                | 0.85             | 0.86                   | 0.83             |
| corr(n_t, y_t) | 0.76      | 0.69                | 0.69             | 0.69                   | 0.69             |
| corr(p_{ht}, y_t) | 0.55      | 0.40                | 0.54             | 0.84                   | 0.40             |
| corr(p_{ht} / r_t, y_t) | 0.57      | 0.35                | 0.47             | 0.77                   | 0.36             |

Autocorrelation

| corr(y_{t,t-1}) | 0.90      | 0.73                | 0.72             | 0.72                   | 0.73             |
| corr(c_{t,t-1}) | 0.83      | 0.73                | 0.73             | 0.73                   | 0.73             |
| corr(i_{t,t-1}) | 0.91      | 0.74                | 0.74             | 0.74                   | 0.74             |
| corr(n_{t,t-1}) | 0.93      | 0.71                | 0.71             | 0.71                   | 0.71             |
| corr(p_{ht,t-1}) | 0.96      | 0.66                | 0.65             | 0.59                   | 0.66             |
| corr(p_{ht,t-1} / r_{t-1}, y_{t-1}) | 0.96      | 0.67                | 0.67             | 0.61                   | 0.66             |

Note: (i) All variables are in logs and HP filtered with a smoothing parameter at 1,600. We simulate the model for 15,000 periods and drop the first 5,000 periods. We then compute sample moments accordingly. We run the simulations 1,000 times and report the sample average.
(ii) Column 2 displays the real business cycle moments from the data.
(iii) Column 3 displays the moments from the benchmark model.
(iv) Column 4 displays the moments from the reference model (without regime switching).
(v) Columns 5 to 7 display the moments from variants for which we shut down collateral quality shocks, housing demand shocks and financial shocks, respectively.
Table 4: Model Fitness

<table>
<thead>
<tr>
<th></th>
<th>log marginal densities of the data</th>
<th>log posterior likelihood (at the posterior modes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>3012.8</td>
<td>3110.1</td>
</tr>
<tr>
<td>Reference</td>
<td>2983.3</td>
<td>3083.9</td>
</tr>
</tbody>
</table>

Note: The reference model denotes the model for which we shut down the regime switching and set the default regime always as the equilibrium one.

Table 5: Estimated Parameters of the Reference Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior Distribution</th>
<th>Posterior Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Distr.</td>
<td>Mean</td>
</tr>
<tr>
<td>$F_1$</td>
<td>Gamma</td>
<td>2</td>
</tr>
<tr>
<td>$F_2$</td>
<td>Gamma</td>
<td>2</td>
</tr>
<tr>
<td>$\psi_X$</td>
<td>Beta</td>
<td>0.5</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Gamma</td>
<td>10</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Gamma</td>
<td>2</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Beta</td>
<td>0.5</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Gamma</td>
<td>0.05</td>
</tr>
<tr>
<td>$\rho_d$</td>
<td>Beta</td>
<td>0.5</td>
</tr>
<tr>
<td>$\rho_h$</td>
<td>Beta</td>
<td>0.5</td>
</tr>
<tr>
<td>$\rho_n$</td>
<td>Beta</td>
<td>0.5</td>
</tr>
<tr>
<td>$\rho_\eta$</td>
<td>Beta</td>
<td>0.5</td>
</tr>
<tr>
<td>$\rho_\theta$</td>
<td>Beta</td>
<td>0.5</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>Beta</td>
<td>0.5</td>
</tr>
<tr>
<td>$\sigma_a$ (%)</td>
<td>Inv. Gamma</td>
<td>1</td>
</tr>
<tr>
<td>$\sigma_h$ (%)</td>
<td>Inv. Gamma</td>
<td>1</td>
</tr>
<tr>
<td>$\sigma_n$ (%)</td>
<td>Inv. Gamma</td>
<td>1</td>
</tr>
<tr>
<td>$\sigma_\eta$ (%)</td>
<td>Inv. Gamma</td>
<td>1</td>
</tr>
<tr>
<td>$\sigma_\theta$ (%)</td>
<td>Inv. Gamma</td>
<td>1</td>
</tr>
<tr>
<td>$\sigma_z$ (%)</td>
<td>Inv. Gamma</td>
<td>1</td>
</tr>
</tbody>
</table>

Note: The posterior distribution is obtained using the Metropolis-Hastings algorithm with 200,000 draws.
Figure 1: U.S. experience of output, house prices and the price-rent ratio. The data sources of output and rents are National Income & Product Accounts (NIPA). House prices are from Federal Reserve, CoreLogic data and National Association of Realtors collected by Bank for International Settlements. The time series are logged and detrended.
Figure 2: Impulse responses to a negative two-standard-deviation collateral quality shock. The solid lines stand for the benchmark model and the dashed lines for the reference model (without regime switching). All the variables are detrended and expressed as percentage deviations from their steady-state values.
Figure 3: Impulse responses to a negative one-standard-deviation housing demand shock. All the variables are detrended and expressed as percentage deviations from their steady-state values.
Figure 4: Impulse responses to a negative one-standard-deviation financial shock. All the variables are detrended and expressed as percentage deviations from their steady-state values.
Figure 5: Historical decomposition of house prices, the price-rent ratio and investment in the estimated model. All series are in deviations from their steady-state values. Shaded areas indicate recessions determined by the National Bureau of Economic Research (NBER).
Figure 6: Model- and data-implied historical paths. The solid lines stand for the benchmark model, the dashed lines for the reference model (without regime switching), and the dotted-dashed lines for the data. Shaded areas indicate recessions determined by the NBER.
Figure 7: Comparison of the model- and data-implied financial tightness. Financial tightness in the data is the net percentage of domestic banks tightening standards for commercial real estate loans released by the Senior Loan Officer Survey from the U.S. Board of Governors of the Federal Reserve System. Shaded areas indicate recessions determined by the NBER.